

University of Mumbai Question Paper
April 2017
Operations Research

Mark – 75

Time – 2 ½

N.B.

1. All questions are compulsory subject to internal choice
2. Figures to the right indicate full marks.

1. Attempt any two from the following:- (15)

- a)** Solve by simplex method
 $\text{Max } Z = 8x_1 + 20x_2$
 Subject to:
 $2x_1 + x_2 \leq 80$...Resources I
 $3x_1 + 4x_2 \leq 96$...Resources II
 $(x_1, x_2) \geq 0$
 Find optimal profit (Max Z)
- b)** ABC Ltd. manufactures two products P and Q. Profit per unit for P and Q is Rs. 40 and Rs. 80 respectively. One unit of P requires 2 machines hours and one unit of Q requires 3 machines hours. Availability of machine hours is 48. Maximum market demand for P is 15 units and for Q is 10 units. Formulate as LPP and solve by graphical method to obtain maximum total profit.
- c) Answer the following:**
- i. Explain the use of slack, surplus and artificial variables in simplex method.
 - ii. Explain what is redundant constraint in graphical solution with the help of a neat sketch.
 - iii. Explain the different types constraints in LPP.

2. Attempt any two from the following:- (15)

- a)** A Company has three factories F_1, F_2 and F_3 with supply of 800, 600, and 1000 units respectively. There are for warehouses W_1, W_2, W_3 and W_4 with demand 400, 500, 700 and 800 respectively. A feasible solution is given below. (With allocations and unit cost data).
- i. Test the solution for optimality using modified distribution method.
 - ii. If the solution is not optimal, find optimal solution by modifying it.
 - iii. Find minimum transportation cost.

Origin	Destination				Supply
	D ₁	D ₂	D ₃	D ₄	
O ₁	5	10	4	5	100
O ₂	6	8	7	2	250
O ₃	4	2	5	7	200
Demand	250	100	150	50	550

To From	W ₁	W ₂	W ₃	W ₄	Supply
F ₁	12 (300)	6 (500)	20	25	800
F ₂	6 (100)	11	15	12	600
F ₃	9	15	17 (200)	7 (800)	1000
Demand	400	500	700	800	2400 2400

- b) There are four machines M₁, M₂, M₃ and M₄. There are five jobs P, Q, R, S and T. Cost of doing each job on each machine is given below (in Rs. Hundred s).

Machine M₂ cannot process job R and Machine M₃ cannot process Job P.

Find optimal assignment of machines and jobs to minimize total cost.

From \ Jobs	P	Q	R	S	T
M ₁	9	11	15	10	11
M ₂	12	9	-	10	9
M ₃	-	11	14	11	7
M ₄	14	8	12	7	8

- c) Answer the following:
- Explain the steps involved in solution of an unbalanced maximization assignment problem.
 - Explain what is meant by degeneracy in a transportation problem and how to resolve it.
 - Explain why Vogel's approximation method (VAM) is better method than North West corner rule (NWCR) to find initial feasible solution of a transportation problem.

Q.3. Attempt any two from the following:

(15)

- a) A small project consists of seven activities.

Activity	Preceding activity	Time(days)
A	-	4
B	-	5
C	-	7
D	A	6
E	B	7
F	C	6
G	D	5
H	E	8
I	F	5

- i. Draw network diagram and find critical path along with project completion time.
- ii. Find earliest latest starting and finishing times of all activities (EST, EFT, LST, LFT).

b) Three time estimates are given for each activity of following project.

Activity	Optimistic (a)	Most Likely (m)	Pessimistic (b)
1-2	6	6	24
1-3	6	12	18
12	12	12	30
2-5	6	6	6
3-5	12	30	48
4-6	12	30	42
5-6	18	30	54

- i. Tabulate expected time (te) and variance of all activities.
- ii. Draw network diagram and find total project completion time (critical path).
- iii. What will be project completion for 90% confidence of completion?

c) Answer the following:

- i. Explain difference between CPM and PERT.
- ii. Explain what is time cost trade off in project crashing is.
- iii. What are the objectives of project crashing?

4. Attempt any two from the following:-

15

a) Four strategic alternatives S_1 , S_2 , S_3 and S_4 are available for countering four states of nature N_1 , N_2 , N_3 and N_4

States of Nature (Events)				
	N_1	N_2	N_3	N_4
Probability	0.2	0.1	0.3	0.4
Strategy S_1	1200	1200	1200	1200
Strategy S_2	1040	1280	1280	1280
Strategy S_3	880	1080	1360	1360
Strategy S_4	700	840	1080	1440

Based on the above information:

- i. Calculate EMV (expected monetary value) for each strategy and find optimal decision.
 - ii. Find EPPI (expected payoff with perfect information) and EVPI (expected value of perfect information).
 - iii. Construct regret table and calculate EOL (expected opportunity loss).
- b) Following payoff matrix refers to a two player game, player A and Player B. Each player has four strategic options.
(Pay off in Rs.)

		Player B			
		I	II	III	IV
Player A	I	500	260	200	210
	II	-50	-100	-40	240
	III	200	400	160	-20
	IV	250	300	100	50

- i. Find the Maximum Strategy.
 - ii. Find the Minimax Strategy.
 - iii. What is the value of the Game?
- c) Five jobs I, II, III, IV and V are to be processed on two machines A and B in the order AB.

Jobs	Processing Time (minutes)	
	Machine A	Machine B
I	90	70
II	40	80
III	40	50
IV	30	10
V	25	35

- i. Find optimal sequence of jobs.
- ii. Find total minimum elapsed time.
- iii. Find idle time for each machine.

Q.5 A company produces two products A and B. Profit per unit for A and B is Rs. 30 and Rs. 50 respectively. Three resources M_1 , M_2 , are utilized. Capacities of M_1 , M_2 , and M_3 are 4, 6, and 12 hours respectively. Following feasible solution has been obtained by simplex method. (15)

Based on the solution answer the following questions.

$C_j \rightarrow$	3	30	50	0	0	0	
C	X	X_1	X_2	S_1	S_2	S_3	B_1
0	S_1	1	0	1	0	0	4
0	S_2	$-\frac{3}{2}$	0	0	1	$-\frac{1}{2}$	0
50	X_2	$\frac{3}{2}$	1	0	0	$\frac{1}{2}$	6
Z_j		75	50	0	0	25	

- i. Is this optimal solutions? Justify.
- ii. What is optimal product mix and optimal profit?
- iii. Is there degeneracy in the solution? Justify.
- iv. Is it unique solution or are there multiple optimal solution? Justify.
- v. What are the shadow prices of M_1 , M_2 and M_3 ?
- vi. Which resources are scarce and which are abundant?
- vii. Find percentage utilization of M_1 and M_2 .

**University of Mumbai Question Paper November
2017**

Operations Research

Mark – 75

Time – 2 ½

Q.1. Attempt any two from the following: (15)

- a. A diet for a sick person must contain at least 18 units of vitamins, 12 units of minerals and 10 units of 7.5 calories. Two foods F_1 and F_2 are available at a cost of Rs. 4 and Rs. 8 per unit respectively. One unit of F_1 contains 6 units of vitamins, 1 unit of minerals and 3 units of calories. One unit of food F_2 contains 1 units of vitamins, 4 units of minerals and 1 unit of calories. Formulate the problem as LPP and solve it graphically to minimize the total cost.

b. **Solve by using Simplex method.**

$$\text{Maximize } Z = 15X_1 + 5X_2$$

Subject to constraints

$$4X_1 + 5X_2 \leq 100$$

$$5X_1 + 2X_2 \leq 80$$

$$X_1 \geq 0, X_2 \geq 0.$$

c. Answer the following:

- i. Explain "Infeasibility" in LPP using graphical sketch.
- ii. Explain limitation of Operations Research.
- iv. What is a unique optimal solution? How do you test of a solution is unique in Simplex method?

Q.2 Attempt any two from the following: (12)

- a. A company is transporting its units from three factories $F_1, F_2,$ and F_3 to four warehouse $W_1, W_2,$

W_3 and W_4 supply and demand of units with transportation cost per unit (in Rs.) are given below.

Cost matrix (Rs. Per unit)

	W_1	W_2	W_3	W_4	Supply (units)
F_1	6	2	6	11	120
F_2	4	12	2	6	95
F_3	10	2	8	16	85
Demand (units)	50	60	90	100	300

- i. Find initial feasible solution (IFS) using Vogel's Approximation method (VAM).
- ii. Test IFS for optimality using MODI method and find optimal solution.
- b. A departmental head has four subordinates who differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate of the time each man would take to perform each task is given in the matrix below. How should the tasks be allocated on one to one basis to minimize the total man- hours? Find optimal assignment of men and tasks.

Time (in hours)

Tasks	Men			
	E	F	G	H
A	1	2	1	1
	8	6	7	1
B	1	2	1	2
	3	8	4	6
C	3	1	1	1
	8	9	8	5
D	1	2	2	1
	9	6	4	0

a. Answer the following:

- i. What is Degeneracy transportation problem? How to resolve it?
- ii. Explain regret matrix in assignment problem.
- iii. How do you detect and find multiple optimal solutions to an assignment problem?

Q.3. Attempt any two from the following: (15)

- a. A small project consists of following six activities.

Activity	Duration (in days)
----------	--------------------

1-2	3
1-3	1
2-5	5
3-4	10
3-5	2
4-5	3

- i. Draw network diagram. Find critical path and project completion time.
- ii. Find EST, EFT, LFT, LST and Total float for all activities.

- b. A projects consists of the activities A,B,C,D,E<F and G whose time estimates and given below:

Activity	Preceding Activity	Estimate duration (in days)		
		(a) Optimistic	(m) Most likely	(b) Pessimistic
A	-	2	4	6
B	-	3	6	9
C	A	1	4	7
D	A	2	3	4
E	B,C	1	3	5
F	D	1	1	1
G	E	1	2	3

- i. Find expected time and variance of all activities.
 - ii. Draw network diagram find expected project completion time.
 - iii. Find probability of project completion in 15 days.
- c. Answer the following:
- i. What is critical path in PERT / CPM analysis?
 - ii. Define optimistic, pessimistic and most likely times in PERT.
 - iii. What is project crashing?

Q.4. Attempt any two from the following: (15)

- a. A company has a choice of adopting any one out of four courses of action A1, A2, A3, A4. There are four possible states of nature S1, S2, S3, S4. Find optimal decision using each of the following decision criterion.
- i. Maximin
 - ii. Maximax
 - iii. laplace
 - iv Hurwicz ($\alpha=0.4$)
 - v. Minimax regret criterion

State of nature	Courses of action			
	A1	A2	A3	A4
S ₁	50	40	30	20
S ₂	50	55	45	35
S ₃	50	55	60	50

S ₄	50	55	60	65
----------------	----	----	----	----

b. Consider the following pay off matrix

		Player B		
		I	II	III
Player A	I	180	60	-90
	II	-100	-40	120
	III	100	80	140

- Find the Maximin Strategy.
- Find the Minimax strategy.
- What is the value of the Game?
- In a factory there are 6 jobs which are to be performed on two machines A and B in the order AB. The processing time (in minutes) for the jobs are given below

Job	Processing time (minutes)	
	Machine A	Machine B
1	30	80
2	120	100
3	50	90
4	20	60
5	90	30
6	110	10

- Determine the optimum sequence for performing jobs.
- Total minimum elapsed time.
- Idle time for each machine.

Q.5. Case Study:

(15)

From four warehouse A,B, C and D the stocks are to be transported to four market are W,X,Y,Z. The supplies from warehouses are 100, 100, 150, 250 tons respectively. Whereas the requirement of markets are 180, 160, 160, 100 tons respectively. The cost matrix (in Rs. per unit) and a solution to the problem is given in the following table.

Markets Warehouse	W	X	Y	Z
A	12	4 (100)	6	16
B	24	26	12 (100)	26
C	16	6 (50)	16	8
D	10 (180)	8 (10)	2 (60)	12

- Is the above solution feasible? justify

- ii. is the above solution non-degenerate? justify.
- iii. is the solution optimal? Find by using MODI method and also find optimal transportation cost.
- iv. Is there more than one optimal solution? justify.
- v. Calculate the opportunity cost to the route A to Y and C to Y.
- vi. What should be the minimum decrease in unit cost of the cell B to X so that the company can optimally use this route alternately?
- vii. If the transport carrier offers discount of 75% on existing cost for the route A to Z should the management accept the offer. Justify.